# CALCULATION OF AIR-FILM COOLING OF A ROTATING DISK

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An analysis and generalization is made of experimental data on the effectiveness of film-cooling of a rotating disk.

Cold-air blowing is often used to prevent overheating of peripheral parts of radial turbine disks. The calculation of such systems of cooling is made difficult by the very limited number of papers devoted to the heat processes taking place in these (see, e.g., [1, 2]). Here, unlike in [1, 2], methods generally proved in the calculation of film-cooling of flat surfaces [3, 7-13] are applied to the solution of these problems.

By analogy with a flat plate, and on the assumption of the presence of an unperturbed main body of the primary stream with uniform parameters at each section of the rotor and a boundary layer along its walls, the effectiveness of thermal protection of the disk walls may be expressed in terms of dimensionless temperature and enthalpy of gas along an adiabatic wall

$$\eta \equiv \frac{i_\delta^0 - i_{\text{a. w}}}{i_1^0 - i_{\text{ox}}^0}. \quad (1)$$

If the law of variation of effectiveness along a radius of the disk

$$\eta = \eta(r) \quad (2)$$

is known, it is possible to calculate its adiabatic temperature and determine the density of local heat fluxes entering a disk subjected to combined cooling as defined in [3] by equation

$$q_{\text{w}} = \alpha_{\text{g}}(T_{\text{a. w}} - T_{\text{w}}). \quad (3)$$

Equations (2) and (3) together with other boundary conditions make it possible to calculate by existing methods (see, e.g., [4]) the temperature levels of a disk. Consequently, in this investigation the attention is, in the main, focused on the determination of the general form of the equation for calculating the effectiveness of film-cooling of disks, followed by experimental definition of its coefficients.

To simplify the flow pattern in the rotor channel we assume that an axisymmetric stream at a Prandtl number close to unity enters the channel without shock and flows through it with a twisting motion in accordance with the law of motion of a solid body, i.e.,

$$V_\varphi = \omega r. \quad (4)$$

The stream motion relative to the disk is then axisymmetric, and its relative velocity is radially directed ($W_\varphi = 0$), while centrifugal forces act in a direction opposite to the motion of the protecting film and their action is equivalent to that of the pressure gradient along the protected surface.

The integral equations defining the two-dimensional nonplanar boundary layer at the adiabatic wall may be written as

$$\frac{1}{r}\frac{d(r\delta^{**})}{dr} + \left[\frac{1}{\rho_\delta}\frac{d\rho_\delta}{dr} + (2+H)\frac{1}{V_{r,\delta}}\frac{dV_{r,\delta}}{dr}\right]\delta^{**} = \frac{C_{f,r}}{2},$$

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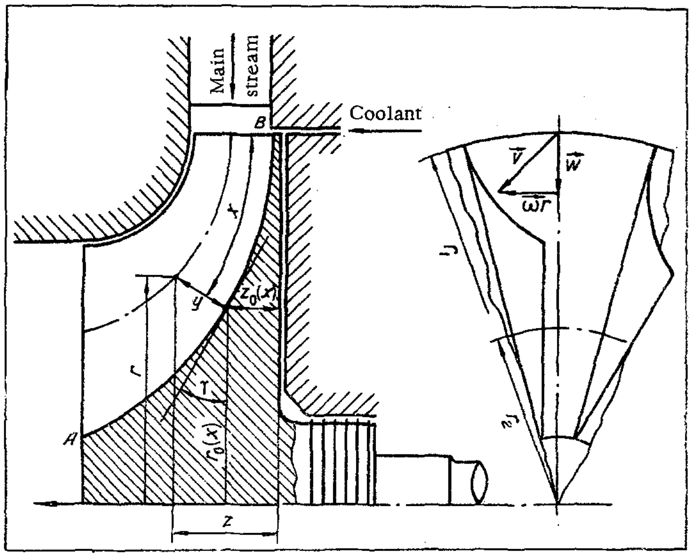


Fig. 1. Diagram of film-cooling of a radial turbine rotating disk.

$$W\frac{1}{r}\frac{d(r\delta_T^{**})}{dr}+\left[\frac{1}{\rho_\delta}\frac{d\rho_\delta}{dr}+\frac{1}{V_{r,\delta}}\frac{dV_{r,\delta}}{dr}+\frac{1}{(i_{a.w}-i_\delta^0)}\frac{d(i_{a.w}-i_\delta^0)}{dr}\right]\delta_T^{**}-\frac{\left(H-\frac{\delta}{\delta^{**}}\right)\delta^{**}}{(i_{a.w}-i_\delta^0)}\frac{di_\delta^0}{dr}=0, \quad (5)$$

where, as usual,

$$\delta^*\equiv\int_0^\delta\left(1-\frac{\rho V_r}{\rho_\delta V_{r,\delta}}\right)dz;\quad \delta^{**}\equiv\int_0^\delta\frac{\rho V_r}{\rho_\delta V_{r,\delta}}\left(1-\frac{V_r}{V_{r,\delta}}\right)dz;\quad \delta_T^{**}\equiv\int_0^{\delta_T}\frac{\rho V_r}{\rho_\delta V_{r,\delta}}\frac{i^0-i_\delta^0}{i_{a.w}-i_\delta^0}dz.$$

If the coordinates

$$r_0=r_0(x) \text{ and } z_0=z_0(x) \quad (6)$$

of the disk protected surface, generated by the rotation of line AB (Fig. 1), are known, it is possible to pass to a new system of coordinates in which the x-coordinate is taken along the generatrix AB and the y-coordinate is normal to it. The transformation formulas are:

$$r=r_0+y\sin\gamma \text{ and } z=z_0+y\cos\gamma, \quad (7)$$

where $\gamma$ is the angle between a tangent to the generatrix and the radial direction.

For small $\gamma$ and $\delta \ll r_0$ Eqs. (5) may be approximated, as in [5], by the expressions

$$\frac{1}{r_0}\frac{d(r_0\delta^{**})}{dx}+\left[\frac{1}{\rho_\delta}\frac{d\rho_\delta}{dx}+(2+H)\frac{1}{U}\frac{dU}{dx}\right]\delta^{**}=\frac{C_{f,x}}{2},$$

$$\frac{1}{r_0}\frac{d(r_0\delta_T^{**})}{dx}+\left[\frac{1}{\rho_\delta}\frac{d\rho_\delta}{dx}+\frac{1}{U}\frac{dU}{dx}+\frac{1}{(i_{a.w}-i_\delta^0)}\frac{d(i_{a.w}-i_\delta^0)}{dx}\right]\delta_T^{**}$$

$$-\frac{\left(H-\frac{\delta}{\delta^{**}}\right)\delta^{**}}{(i_{a.w}-i_\delta^0)}\frac{di_\delta^0}{dx}=0, \quad (8)$$

derived by the substitution of v for $V_z$, $-u$ for $V_r$, $-U$ for $V_{r,\delta}$, $-C_{f,x}$ for $C_{f,r}$, and the derivatives $\partial/\partial y$ for $\partial/\partial z$ and $-\partial/\partial x$ for $\partial/\partial r$.

Assuming in the general case the friction and heat transfer laws in the turbulent boundary layer to be such [6] that

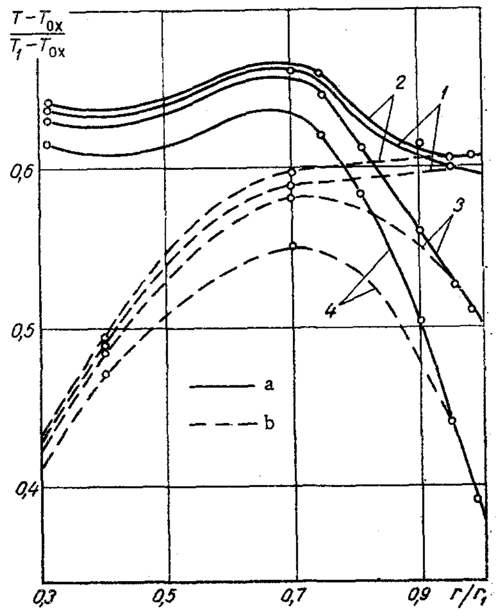


Fig. 2. Experimentally determined temperature distribution along the disk surface (curves a and b relate to the hot gas and cooling air sides, respectively) with the cooling air injected through the radial gap. 1, 2, 3, and 4 denote test runs.

$$\frac{C_{f,x}}{2} = \tilde{A}\mathrm{Re}^{**^{-m_1}}, \qquad \mathrm{St} = \tilde{A}\,\mathrm{Re}_m^{**^{-m_1}}, \tag{9}$$

and using the Stepanov–Mangler type of change of variables [5]

$$\bar{x} = \int_{x_0}^{x} r_0^{(1+m_1)}(\xi)\,d\xi;\quad \bar{y} = r_0 y;\quad \bar{u} = u;\quad \bar{U} = U;\quad \bar{v} = \frac{v}{r_0} + \frac{r_0'}{r_0}\frac{y}{r_0}u;\quad \bar{p} = p;\quad \bar{i} = i \text{ and } \bar{i}_\delta^0 = i_\delta^0, \tag{10}$$

we reduce Eqs. (8) to the form

$$\frac{d\bar{\delta}^{**}}{d\bar{x}} + \left[\frac{1}{\bar{\rho}_\delta}\frac{d\bar{\rho}_\delta}{d\bar{x}} + (2+\bar{H})\frac{1}{\bar{U}}\frac{d\bar{U}}{d\bar{x}}\right]\bar{\delta}^{**} = \frac{\bar{C}_{f,\bar{x}}}{2},$$

$$\frac{d\bar{\delta}_T^{**}}{d\bar{x}} + \left[\frac{1}{\bar{\rho}_\delta}\frac{d\bar{\rho}_\delta}{d\bar{x}} + \frac{1}{\bar{U}}\frac{d\bar{U}}{d\bar{x}} + \frac{1}{(\bar{i}_{\mathrm{a.\,w}} - \bar{i}_\delta^0)}\frac{d(\bar{i}_{\mathrm{cт.a}} - \bar{i}_\delta^0)}{d\bar{x}}\right]\bar{\delta}_T^{**} - \frac{\left(\bar{H} - \frac{\bar{\delta}}{\bar{\delta}^{**}}\right)\bar{\delta}^{**}}{(\bar{i}_{\mathrm{a.\,w}} - \bar{i}_\delta^0)}\frac{d\bar{i}_\delta^0}{d\bar{x}} = 0. \tag{11}$$

The system of equations (11) which defines the axisymmetric nonplanar boundary layer along the working surface of a rotating disk is of the same form as that defining the development of the boundary layer along an adiabatic plate. Hence it is possible to use data on the variation of (cooling) efficiency along a flat plate for evaluating the effectiveness of thermal protection of a radial turbine disk.

Investigations carried out at the Institute of Technical Thermophysics of the Academy of the Ukrainian SSR [9-12] had shown that in the case of a uniform profile of total enthalpy in the main flow upstream of a single tangential slot the effectiveness (of cooling) downstream of such a slot is given by the following equation (Fig. 3):

$$\bar{\eta} = f(\bar{A}) = \begin{cases} 1 & \text{for } \bar{A} \leqslant 3, \\ \left(\frac{1}{3}\bar{A}\right)^{-0,3\,n_i} & \text{for } 3 \leqslant \bar{A} \leqslant 11, \\ \left(\frac{1}{7.43}\bar{A}\right)^{-n_i} & \text{for } \bar{A} \geqslant 11 \text{ and } n_i = 0.95, \end{cases} \tag{12}$$

where $\bar{A} \equiv \bar{\Delta}^{+1.5}\overline{\mathrm{Re}}_1^{-0.25}\bar{m}^{-1.3}\Theta^{-1.25}(x/s)$ is a semiempirical generalizing dimensionless complex (parameter).

As shown in [13], acceleration of the main stream affects only slightly the effectiveness of thermal protection of a flat wall. Hence the effect of the radial gradient of velocity $V_{r,\delta}$ on the efficiency of film-cooling of a rotating disk can, in the first approximation, be neglected.

TABLE 1

| Test run | $G_1$, kg sec | $G_{ox}$, kg sec | $T_1$, °K | $T_{ox}$, °K | sec$^{-1}$ | m |
|---|---|---|---|---|---|---|
| 1 | 0,1475 | 0,00200 | 773 | 400 | 1540 | 0,46 |
| 2 | 0,1800 | 0,00188 | 773 | 445 | 1885 | 0,32 |
| 3 | 0,1800 | 0,00380 | 773 | 437 | 1885 | 0,65 |
| 4 | 0,1800 | 0,00776 | 773 | 423 | 1885 | 0,69 |

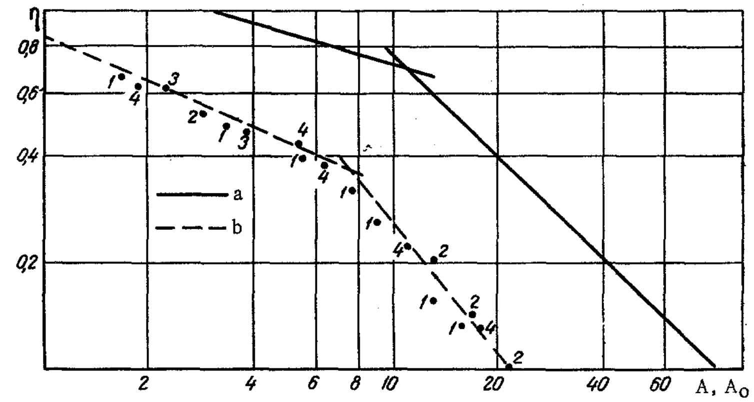


Fig. 3. Dependence of the effectiveness of film-cooling of a flat surface (curve a, Eq. (12)) and of a disk (curve b, Eq. (6)) on the dimensionless complex (parameter) A. 1, 2, 3, and 4 denote experimental readings at four operation conditions.

If, furthermore, the total enthalpy variation $i^0_\delta$ of the main stream is assumed to be linear throughout the length of the rotor working channel, then Eq. (12) with Eqs. (11) can be used for calculating the effectiveness of film-cooling of radial turbine disks.

In this case we have

$$\bar{\eta} = \eta;\ \bar{\Delta} = \Delta;\ \overline{\mathrm{Re}_1} = \mathrm{Re}_1 r_0;\ \bar{m} = m;$$
$$\bar{\Theta} = \Theta;\ \bar{s} = s r_0 \text{ and } \bar{x} = \int_0^x r_0^{(1+m_1)}\,dx, \tag{13}$$
$$\text{where } r_0 \cong r_1 - x \text{ and } m_1 = 0.25,$$

and Eq. (12), when applied to an axisymmetric flow through a disk, becomes

$$\eta \equiv \frac{i^0_\delta - i_{\text{a. w}}}{i^0_1 - i^0_{\text{ox}}} = f(A_0), \tag{14}$$

where

$$A_0 \equiv \Delta^{+1.5}\mathrm{Re}_1^{-0.25} m^{-1.3}\Theta^{-1.25}\,\frac{r_1 - x}{2.25 s}\left[\left(\frac{r_1}{r_1 - x}\right)^{2.25} - 1\right]. \tag{15}$$

Thus the experimental determination of the effectiveness of film-cooling of rotating disks must be aimed at finding the functional relationship (14). With this in view, measurements were made on the experimental stand of the Institute of Technical Thermophysics of the Academy of Sciences of the UkrSSR of the surface temperature of a 140 mm diameter disk of the radial turbine of a type TKP-14-2 production turbocompressor cooled by air injected into the gap between the nozzle ring and the rotor. Measurements were taken at 19 points along the disk radius.

Tests were run at four operation conditions (Fig. 2) with the initial gas temperature of 823°K and other parameters as defined below:

$$50\cdot 10^3 \geqslant \mathrm{Re}_1 \frac{x}{s} \geqslant 35\cdot 10^3;\ 0.7 \geqslant m \geqslant 0;\ 0.61 \geqslant \Theta \geqslant 0.25.$$

Contrary to [3, 7-13], where the temperature distribution was investigated along heat-insulated flat surfaces, it is not possible in this case to determine experimentally the adiabatic temperature $T_{a.w}(r)$ at the disk surface owing to the heat flow within the disk and to heat transfer to the air flowing past its rear face. This results in the measured temperature $T_w(r)$ of the disk surface always being lower than the adiabatic temperature $T_{a.w}(r)$.

In this investigation $T_{a.w}(r)$ (i.e., the effectiveness $\eta$ of the disk film-cooling) was determined by solving the converse heat-conduction problem using an electrical grid model on an ÉI-12 integrator. The same model was used for determining the heat transfer coefficients along the hot-gas side from data obtained in experiments without film cooling ($m = 0$), as suggested in [14].

In accordance with the above exposition, parameter $\alpha_g(r)$ was assumed to be independent of the presence and intensity of (coolant) injection (parameter m). Boundary conditions, obtained experimentally from measurements of the temperature of metal, were set down for the computations with the aid of the electrical model.

The error in the calculation of the effectiveness of film-cooling of the rotating disk, estimated in the generally accepted manner (see, e.g., [5]) described above, does not exceed 8-12% for an error of an order of 2.5% in measurements of the rotating disk temperature.

The dependence of experimentally obtained values of the effectiveness of film-cooling of rotating disks in the complex (parameter) $A_0$ is given in Fig. 3 for $\Delta^{1.5} = 1$. This diagram shows that experimental values of effectiveness are distributed along a certain mean line with a scatter not exceeding ±15%, which is in agreement with the results of the present theoretical analysis. An approximation of the separate segments of the mean line by power functions yields for the effectiveness an empirical equation of the form

$$\eta = \begin{cases} \left(\frac{1}{0.66}\frac{A_0}{\Delta^{1.5}}\right)^{-0,3n} & \text{for } 1.5 \leqslant A_0 \leqslant 7.3, \\ \left(\frac{1}{3.26}\frac{A_0}{\Delta^{1.5}}\right)^{-n} & \text{for } 7.3 \leqslant A_0 \text{ and } n = 1,\ 2. \end{cases} \quad (16)$$

The curve of the effectiveness of thermal protection, appearing in [7-13] and defined by Eq. (12), pertaining to the injection of air through a tangential slot in the flat surface is also plotted in Fig. 3. The difference between the values of effectiveness derived by the present analysis and those obtained for flat surfaces is due to the injection of coolant into the working part of the tested centripetal turbine and, possibly, also to the neglect of the initial and thermal boundary layers of the primary flow upstream of the gap, when processing experimental data.

Until it becomes possible to formulate specific recommendations for the assessment of the effect of the slot shape and of the coolant inlet angle, and, also, the completion of detailed investigations into the effect of the initial and thermal boundary layers, Eq. (16) may be used for evaluating the effectiveness of film-cooling of rotating disks similar to that investigated here as regards conditions of coolant injection and geometric dimensions.

The present investigations permit the formulation of the following fundamental conclusions:

1) If the stream flowing through a disk is defined by the equation $\omega r = \text{const}$, the method for determining the effectiveness of cooling flat surfaces may be used for calculating the effectiveness of film-cooling of rotating disks with recourse to the coordinate transformation (11).

2) In the case of disks dimensionally similar to the above described, and for conditions of flow close to those of our experiments, the assessment of film-cooling effectiveness may be made, in the first approximation, by using the empirical equation (16) derived from these experiments.

3) In order to evaluate the nonadiabaticity of surfaces to be protected it is expedient in investigations of film and combined cooling to recalculate related experimental data, using the methods and facilities of electrical modelling of heat conduction problems.

## NOTATION

| | |
|---|---|
| r, $\varphi$, and z | are cylindrical coordinates; |
| x, y, and $\gamma$ | are curvilinear coordinates (see Fig. 1); |

| | |
|---|---|
| $\rho$, T, and $i^0$ | are, respectively, the density, the temperature, and the total enthalpy of the stream; |
| V and W | are, respectively, the absolute and the relative velocities of the main flow through the disk channel (Fig. 1); |
| u and v | are the projections of the flow velocity on the axes of the system of curvilinear coordinates (x, y); |
| U | is the projection of the velocity of the unperturbed working stream on the x-axis; |
| $\omega$ | is the angular velocity of rotation; |
| $\delta$, $\delta^*$, $\delta^{**}$, and $\delta_T^{**}$ | are the corresponding thicknesses of the boundary layer; |
| $H \equiv \delta^*/\delta^{**}$ | is the form factor; |
| $C_f$ | is the coefficient of surface friction; |
| q | is the density of the heat flux; |
| $\alpha_g$ | is the coefficient of heat transfer; |
| $\eta$ | is the effectiveness (of cooling); |
| $s_H$ | is the height of the slot (gap); |
| $s = s_H + 0.5h_{kp}$; | |
| $Re_1 \equiv \rho_1 u_1 s/\mu_1$ | is the Reynolds number; |
| $St \equiv \alpha_g/\rho_1 u_1 c_{p1}$ | is the Stanton number; |
| $m \equiv \rho_{ox} u_{ox}/\rho_1 u_1$ | is the coefficient of (coolant) injection; |
| $\Theta = T_{ox}/T_1$ | is the temperature factor; |
| $\Delta \equiv (s_H + h_{kp} + \delta^*(0))/(s_H + 0.5h_{kp})$; | |
| A | is a constant; |
| $m_1$, $n_i$, and n | are exponents in Eqs. (9), (12), and (16), respectively; |
| G | is the mass-flow rate. |

## Subscripts

| | |
|---|---|
| 1 | denotes parameters of the unperturbed flow upstream of the gap; |
| ox | denotes parameters of the cooling stream; |
| w | denotes disk wall; |
| a. w | denotes the adiabatic wall; |
| $\delta$ | denotes parameters of the boundary layer; |
| r and x | denotes projections on the corresponding coordinate axes; |
| $\bar{A}$, $\bar{\delta}$,... | denotes parameters of the design and the "flat" boundary layer. |

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